Section 6.3 Solutions

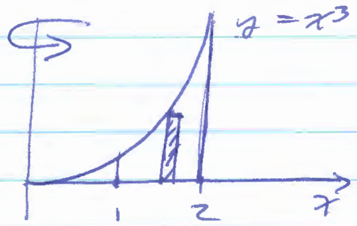
Math 31

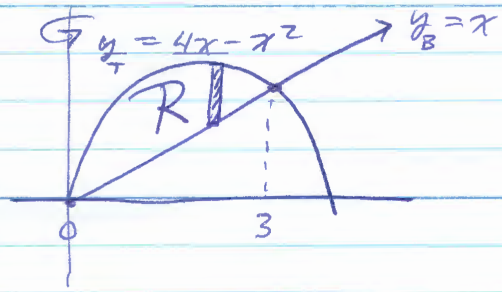
4. $V = \int_1^2 2\pi x y\, dx$

$= \int_1^2 2\pi\, x \cdot x^3\, dx$

$= 2\pi \int_1^2 x^4\, dx = 2\pi \left[\frac{1}{5}x^5\right]_1^2$

$= \boxed{\frac{62}{5}\pi}$





6. $V = \int_0^3 2\pi x (y_T - y_B)\, dx$

$= 2\pi \int_0^3 x\left[(4x - x^2) - x\right] dx$

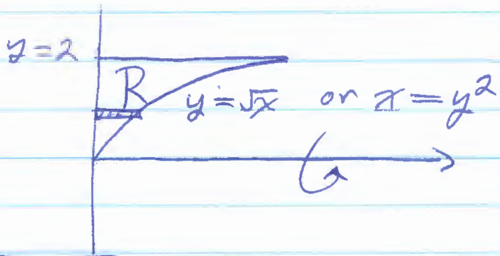
$= 2\pi \int_0^3 x\left[3x - x^2\right] dx = 2\pi \int_0^3 (3x^2 - x^3)\, dx$

$= 2\pi \left[x^3 - \frac{1}{4}x^4\right]_0^3 = \boxed{\frac{27}{2}\pi}$

10. $V = \int_0^2 2\pi y\, x\, dy$

$= 2\pi \int_0^2 y \cdot y^2\, dy$

$= 2\pi \int_0^2 y^3\, dy = \frac{\pi}{2} y^4 \Big]_0^2 = \boxed{8\pi}$

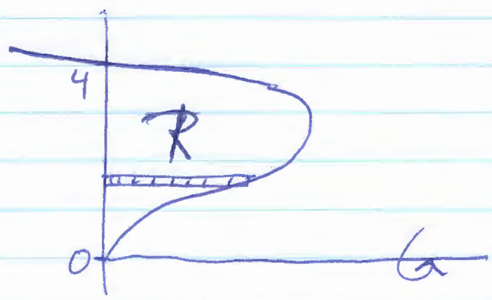


12. $V = \int_0^4 2\pi y\, x\, dy$

$= 2\pi \int_0^4 y(4y^2 - y^3)\, dy$

$= 2\pi \int_0^4 (4y^3 - y^4)\, dy$

$= 2\pi \left[y^4 - \frac{1}{5}y^5\right]_0^4 = \boxed{\frac{512}{5}\pi}$



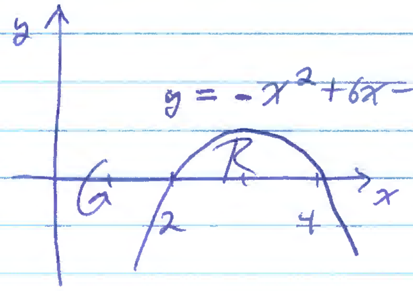
Section 6.3 Solutions (continued)

38. The problem can be simplified by noting that the volume will be the same as that obtained by rotating the region R between the curves $y = 1 - x^2$ and $y = 0$ about the x-axis. (why?). Then, using disks and symmetry, the volume is given by

$$V = 2\int_0^1 \pi y^2\,dx = 2\pi \int_0^1 (1-x^2)^2\,dx$$

$$= 2\pi \int_0^1 (1 - 2x^2 + x^4)\,dx$$

$$= 2\pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_0^1 = \boxed{\frac{16}{15}\pi}$$



Equivalent problem